Joistings 25

## Rescuing Quantum Mechanics

This essay was originally altogether more complex. Indeed, ti was beginning to look more like a book, mainly a detailed commentary of Volume 3 of Feynman's Lectures on Physics. What follows was previously the third of a penultimate draft containing four sections dealing with the problem and with my problem of presentation. That previous draft contained a chapter by chapter commentary on Feynman's 21-chapter volume. Might I say briefly and helpfully what my problem was, and is? For I have not solved it.

The solution lies in the future, in a developed functional specialization that will place my muddied "Interpretation of Feynman Volume 3" in the full swing of a mature cyclic physics. It is not just that my interpretation is muddied: it is that it is trapped, herenow between us, in a culture of communication and of physics that is a shambles. That, of course, you find hard to believe. If you are a contemporary physicist, you find it unacceptable. If you are in the business of pop-physics - whether you are Hawking or Green or Davies - you leap from the conviction that we are doing quite nicely in physics to a destructive science fiction, a fiction about human understanding and about the elusive beings of physics. But I should halt immediately: you rightly smell another book. Have a preliminary look, if you like, at note 30 below. Let us begin, then: and I am eccentric enough to retain the reminder of the full context of the 'disappeared' other sections $1,2,4$, by holding to the number three for the "central section" of this Joistings!

## 3. "No Matter; Try Again; Fail Again; Fail Better" ${ }^{1}$

This final effort has been perhaps sufficiently contextualized now by the previous Joistings 24, and the previous sections here. So let me get to the job of giving some pointers that would help forward the serious reader of volume three of the Feynman lectures towards sublating Feynman's view into an anticipation of an adequate quantum mechanics. By the serious reader I mean someone who has got down to the task the way Lonergan suggests one get down to the task of creatively reading Aquinas. ${ }^{2}$

My reader, at all events, is struggling to understand in a way that would carry him or her beyond Feynman towards what I call The Standard Model (of 2111) ${ }^{3}$, quite beyond the present mess. ${ }^{4}$ Feynman, battling gallantly forward, is part of that mess of history, so I might help further, encourage patience further, by noting that his twenty chapters with the Epilogue of "A Seminar" is in a way much tougher than the twenty chapters and Epilogue of Insight. In a way: for, Lonergan in Insight has both shaken off
${ }^{1}$ I am quoting Samuel Beckett from memory. No doubt someone will help me out here with a precise reference.
${ }^{2}$ I am referring to the Epilogue reflections in the famous Verbum articles, Lonergan, Word and Idea in Aquinas, University of Toronto Press, 1997. A short text from page 223 should jog the memory. "Only by the slow repetitious circular labor of going over and over the data, by catching here a little insight and there another, by continuous adjustments and cumulative changes of one's initial suppositions and perspectives and concepts can one hope to attain the development of one's own understanding as to hope to understand what Aquinas understood and meant" and what Feynman understood and meant.
${ }^{3}$ I have appealed regularly in the past decade to a fruitful parallel between Lonergan's model of global inquiry and the Standard Model that dominated chromodynamics at the end of the twentieth century. See Lonergan’s Standard Model of Effective Global Inquiry., available on the website www.philipmcshane.ca from January 2007. Why the date 2111? See note 24 below.
${ }^{4}$ On the mess, see note 30 below.
the chains of conceptualism and put on the armour of empiricality in such a manner as to take him out of twentieth century. In a way, then, Feynman is tougher work. Still, I would note that a relative mastery of Insight is required to serious follow through on my hints.

Now you may recall that I already faced a parallel task, in the Cantowers, with regard to Insight. I am referring in particular to those Cantowers that parallel chapters of Insight, like 14-21. What was my "Try Again" there? Consider chapter 17 of Insight and its parallel Cantower 17. A serious commentary on Insight 17 would be a very large book, so my effort consisted in commenting on selected bits and pieces. Did the comments help? There was no follow up, so I cannot say. But I did try again and failed better in chapter 9 of ChrISt in History. ${ }^{5}$ Here, at 75, there is unlikely to be such a followup with a better failure.

So I proceed by selecting for comment bits and pieces of four chapters of Feynman. I comment with brutal brevity: F7, F8, F16, F20. Indeed, my comment in the text below is really only on a single page of $\mathbf{F 8}$ : the others are merely given pointermention in the notes. It is quite clearly a matter of brutal brevity and I frankly enjoy Fermat's marginal comment, ".... cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet," ${ }^{6}$ on regard to not only Feynman but also the

[^0]larger project of reaching an eschatological contextualization of physics. More prosaically I recall four unwritten Cantowers, which would have lifted a serious reader forward regarding quantum mechanics. Here we have only a piece of one Joisting. "No Matter".

In the previous Joisting I asked you to struggle with the first three chapters of Feynman III, with an eye on the meaning both of probability and energy, and you were discomfortingly thrown back there to my own earlier struggles with both these topics. Might I presume that you carried forward similarly on those two topics through the other 18 chapters of Feynman? Perhaps the you that reads this now is a you of 2106, A Wiles with ten years work behind him or her, amused at my marginality?

Let us then skip to the third section of F8. The title question is "What are the base states of the world?"Perhaps the title reminds you a little of Laplace and the possibility of a deductive determinism, and the reminding is useful. But now we are better off than Laplace: we have better mathematics, better physics, better techniques and symbolisms. We have - easily fitting into a margin - $\mathbf{H}_{\mathrm{ij}}$. What might you mean, and what do I mean by $\mathbf{H}_{\mathrm{ij}}$ ? Of course, what Feynman means nudges both of us along, and what he means carries us way beyond this struggle to teach a second year university class. Indeed, it may very well have carried you forward to push for a better meaning than Feynman of Dirac's strange suggestion that leads to associating the task of "getting from" the state at $x, y, z, t$ to a neighboring future (again, recall Laplace) with an exponential function of S. ${ }^{7}$

[^1]Let us stay with the elementary text. You might simplify - but dangerously - by thinking of just of "getting from" $t$ to $t+d t$. It is handy to have before us here two key paragraphs of Feynman.
"The idea, then, is that to describe the quantum mechanical world we need to pick a set of base states I and to write the physical laws by giving the matrix of coefficients $H_{i j}$. Then we have everything - we can answer any question about what will happen. So we have to learn what the rules are for finding the $H^{\prime}$ 's to go with any physical situation - what corresponds to a magnetic field, or an electric field, and so on. And that is the hardest part. For instance, for the new strange particles, we have no idea what $H_{i j}$ 's to use. In other words, no one knows the complete $H_{i j}$ for the whole world. (Part of the difficulty is that one can hardly hope to discover the $H_{i j}$ when no one even knows what the base states are!) We do have excellent approximations for nonrelativistic phenomena and for some special cases. In particular the forms that are needed for the motions of electrons in atoms - to describe chemistry. But we don't know the full true $H$ for the whole universe.

The coefficients $\boldsymbol{H}_{i j}$ are called the Hamiltonian matrix, or, for short, just the Hamiltonian. (How Hamilton, who worked in the 1830's, got his name on a quantum mechanical matrix, is a tale of history.) It would be better called the energy matrix, for reasons that will become apparent as we work with it. So the problem is: know your Hamiltonian!" ${ }^{8}$
worthwhile for you to push towards grasping the relationship between them. Follow up note 11 below.
${ }^{8}$ F8, page 10. It would be a large distraction to develop that slogan in relation to the Einstein's or Bell’s paradoxes. You need to seriously follow up the pointers given by Feynman in F18, pp. 8-9. Y ou have to get to a precise grip on the entire section 3 there, "The Annihilation of the Positron". But the full context requires a sublated version of Feynman's book. E.g. to

Those last two sentences of Feynman give us a great lead. Yes, know your
Hamiltonian, but now know can have, at its best, all the twistedness of that symbol (about) ${ }^{3}$ that I introduced in various places. ${ }^{9}$ Let us, oh so briefly, push it within the scope of the second definition of generalized empirical method. ${ }^{10}$ Then, yes, it is better called the energy matrix. ${ }^{11}$ In each hamiltonian case it is a matrix, and I must presume that you have somehow these cases "in your paws"12 from repeatedly struggling with
correct his view that "the principles of quantum mechanics are not only interesting, but so deep that by adding only a few extra hypotheses about the structure of space, we can deduce many properties of physical systems" (F6, p. 2). One must push towards precision about space-time being, not some container, but constituted by the conjugates of things. A helpful key in your reflections is the conclusion of section 4 of $\mathbf{F 1 6}$ : "If there are two particles in nature which are interacting, there is no way of describing what happens to one of the particles by trying to write down a wave equation for it alone. The famous paradoxes that we considered in earlier chapters when the measurements made on one particle were claimed to be able to tell what was going to happen to another particle, or were able to destroy an interference - have caused people all sorts of trouble because they have tried to think of the wave functions of one aprticle alone, rather than the correct wave function in the coordinates of both particles. The complete description can be given correctly only in terms of functions of the coordinates of both particles"(p. 11 of F16).
${ }^{9}$ See, for example, the most recent presentation in section 2 - titled "(about) ${ }^{3}$ " - of chapter 2 of ChrISt in History (available on the website).
${ }^{10}$ This is presented briefly in Joistings 21, but it is given in Lonergan in A Third Collection, the top lines of page 141 . Joistings 21 pushes its meaning towards the fullness of the third and fourth definitions of generalized empirical method.
${ }^{11}$ The connection with the "usual" S-matrix is developed on pages 8-9 of F8: the limiting case of the "change of state operator, $\mathrm{U}\left(\mathrm{t}_{2}, \mathrm{t}_{1}\right)$ as the two ts are taken infinitely back and forward ( page 8.4: you are to think here of a scattering problem: the out-of-range-at the two ends states). Then (page 9.6) identifies H : "the terms of $H_{i j}$ are just the derivatives with respect to $\mathrm{t}_{2}$ of the coefficients $\mathrm{U}_{\mathrm{ij}}\left(\mathrm{t}_{2}, \mathrm{t}_{1}\right)$ evaluated at $\mathrm{t}_{2}=\mathrm{t}_{1}=\mathrm{t}$ ". Following that up in the Feynman book mentioned in note 7 is a larger challenge!
${ }^{12}$ I am connecting in here Lonergan's comment on control of meaning in Phenomenology and Logic, page 357, where the topic is Euclidean geometry. A text of broader significance in the matter of control is The Ontological and Psychological Constitution of Christ, 151, which I have quoted regularly these past decades. It is part of the metagram W3, where the reference is to the Latin text, De Constitutione Christi, at page 80.
the whole 21 chapters of Feynman III. That pawhold, with the other paw of selfattention, lifts you towards the reading that "I have in mind" of the first Feynman paragraph I quoted. Does the hamiltonian give "the physical laws"? Get back to our lead into the Feynman quotation: the matter of getting from $t$ to $t+d t$.

Now I would have you brood sufficiently over the matrix to get you away from standard perspectives like "transition probabilities" to a perspective that would grasp the hamiltonian as relating $t$ to $t+d t$ in a fresh way. You may think of the relating as a type of rotation, but it is a peculiar rotation in a peculiar symbolic space. ${ }^{13}$ The important twist is the rotation, the twist, the self-rotation, towards the meaning of each element of the hamiltonian, each matrix element. A matrix is an originating pattern, and here the origination is an asymmetric ${ }^{14}$ pattern that correlates sequential patterns -

[^2]massively interlocked with other patterns - of energy formation.
Here I find brevity the only way to go, for the moment. It is either brevity or a book: do I again bring to mind the Fermat problem of giving a marginal note?! In energy, "in potency there are at least two aspects of its proper contribution to the constitution of proportionate being, and, on the other hand, its relation to the other contributions of form and act. ${ }^{15}$ So here there is a relating of two proximate space-time
statistical probability of the state of the world which exists." We are back at the complex context that I mentioned in note 32 of Joistings 24. Another context that I would add here, though it is not available to me, is that of Feynman's Ph.D. thesis of 1942 in Princeton: The Principle of Least Action in Quantum Mechanics.
${ }^{15}$ Insight, 450[476]. Here again I would suggest that you add in the topic of entropy, and indeed the topic of negentropy (I am thinking here of Schrödinger's popular lectures given in Trinity College Dublin, titled What is Life?).It would have been a topic in the Cantowers that followed the four (42-45) on Quantum Mechanics: Cantower 46, "Energy and Entropy" and Cantower 47, "Heuristic Thermodynamics". This is a zone that was not developed by Lonergan and I suspect that he did not have the time and 'energy' to intussuscept and sublate the relevant sections of Lindsay and Margenau, Foundations of Physics. One would best, now, add an up-to date context such as that pointed to by Ian Lawrie, A Unified Grand Tour of Theoretical Physics, ICP publishing, 1998 pb, chapter 10.
sets of actual-possibilities ${ }^{16}$ of potency, of energy. ${ }^{17}$ This, I hope, brings you back to thinking about the suggestion of the previous Joistings, the apparently simple suggestion of replacing the word amplitude with aptitude. ${ }^{18}$ But now perhaps you are in a position to add and twist forward ${ }^{19}$ a larger context: Bell's pointers regarding beables; ${ }^{20}$

[^3]${ }^{18}$ See Joistings 24, in the paragraph leading up to note 32.
${ }^{19}$ Perhaps you are, later in this century. In the longer cycle of incline there is the process of page 250 of Method in Theology, with its discomforting twists of such views as are referred to in the following notes, with the twist especially toward a self-attention that asks the historian of physics to be "at pains not to conceal his tracks but to lay his cards on the table"(Method in Theology, 193), but the non-concealing pain is the grim bone-climb to metaphysical equivalents of, literally, the tracks described and explained in the history of physics and concealed precisely by the biased language of those accountings: we find ourselves in the discomforting task of note

Redhead's recalling Aristotle writing of form; ${ }^{21}$ Bub's reach for refinements of Bohme's
hidden variable view. ${ }^{22}$ The main difficulty is to lift your own thinking, at whatever
level it is at, towards the "(about) ${ }^{3 "}$ that is involved in the "come about." ${ }^{23}$ But the full
lifting that would solve the problem of history in physics and physics in history, is a

30 below.
${ }^{20}$ I am thinking here of the title of one of Bell's essays, "The Theory of Local Beables", one of many round the topic in J.S.Bell, Speakable and unspeakable in quantum mechanics: collected papers in quantum mechanics, Cambridge University Press, 1987. There is a further volume regarding Bell's work that you might find useful as context: John Ellis and Daniele Amati, Quantum Reflections, Cambridge University Press, 2000.
${ }^{21}$ Chapter 2 of Michael Redhead, Incompleteness, Nonlocality and Realism. A Prologomenon to the Philosophy of Quantum Mechanics, Clarendon Paperback, 1992, gives a good account of the various views. I quote a relevant passage here: "The idea of potentiality was central to Aristotelian physics - crudely, that the acorn 'possessed' the potentiality of becoming an oak tree, and that all change consists just of the actualization of potentialities. Heisenberg, in his later writings on the philosophy of QM, was particularly concerned to stress the Aristotelian affinities of this type of interpretation. Another was of expressing this view is the concept of 'latent' quantities dues to Margenau, which he contrasted with 'possessed' quantities considered in classical physics. Measurement of an observable not an eigenstate of that observable is supposed to convert latent values into possessed values."( p.48)
${ }^{22}$ Jeffrey Bub, Interpreting the Quantum World, Cambridge University Press, pb,1999, is a more complex and detailed discussion of the problem. Chapter 6 begins with a discussion of Bohmian mechanics, and then goes on to treat of "the modal interpretation": "The idea behind a 'modal' interpretation of quantum mechanics is that quantum states, unlike classical states, constrain possibilities rather than actualities"(p.173). The dominant present ethos is, of course, that of the Copenhagen Interpretation: a complex of muddles that I touch on only generically in this short essay.

[^4]task for "a hundred years or so". ${ }^{24}$ The larger lifting and the larger context is the global cyclic division of labour described elsewhere. ${ }^{25}$

Still, I can lift forward this marginal note a little in various ways, indeed in many ways that bubble up and send me off on a silly optimism of possible communication. ${ }^{26}$ After a day's brooding pause, however, I settle for a few twisting footnote comments round the heart of two of Feynman's chapters: "The Dependence of Amplitude on Time" and "The Dependence of Amplitude on Position." ${ }^{27}$

The comments require your struggle with the manner in which Feynman, the skilled pedagogue, reduces "the world" conveniently to a focus on little worlds with lesser states, particularly to a focus on two-state systems. ${ }^{28}$ But he starts in a manner that is worth following up in the context of the problem of F7, with "a system for which only one base state is required for the description; it is an approximation we could make

[^5][^6]for a hydrogen atom at rest, or something similar." ${ }^{29}$ So I would point you towards puzzling towards asking about the similarity between the hydrogen atom and the universe, the universe of now or the universe of 13.7 billion years ago. My footnote merely adds a few further pointers that nudge you towards seeing the deep flaws in Feynman's heuristic, common cancers of contemporary physics. ${ }^{30}$

Finally, swing to the conclusion of that quite brilliant chapter $20 .{ }^{31}$ Eventually,
${ }^{29}$ F8-10.
${ }^{30}$ What might I write here that would lift you beyond impressionistic hints? Perhaps start by reflection on the flaws lurking in the statement of the first page of $\mathbf{F 7}$ :"An electron alone in empty space can, under certain circumstances, have a definite energy". Where is Feynman coming from here? Etc, etc. Might we spin that electron like Brian Green spins Newton’s bucket in The Fabric of the Cosmos: Space, Time and the Texture of Reality ?(Knopf, New York, 2006: see the index there, under bucket: compare this mess with Lonergan's brief treatment in Insight). I could add a solid list of experts from Einstein to Bell and Feynman muddled about space-time, especially as they talk of those terrible traveling twins. You get a larger perspective on the muddle from the recent Wheeler-memorial volume, Science and Ultimate Reality. Quantum Theory, Cosmology and Complexity, edited by John Barrow, Paul Davies and Charles Harper, Cambridge University Press, 2004. Especially check out the utter shambles of Part VI, on Emergence. On emergence and aggreformism see Cantower 29, against the background of Insight and Randomness Statistics and Emergence. I would note that you are up against "the problem of interpretation" as it is first posed, in chapter 5 of Insight. But you have the larger challenge of pushing for metaphysical equivalents of Feynman's statements, or anyone elses. A huge job in the transposition of physics in this century. I would note also that there is the question of a large book supplementing the brief treatment of measurement given in Insight chapter 5. Room here, certainly, for a cheeky Fermat-margin comment! But, seriously, I think it should be evident that the thinking about things and their couplings in present physics is dominated by the cloudy business of "bodies". Is radiation a spread of bodies of little bodies? And so on. See the final footnote of this Joistings.
${ }^{31}$ You might begin by noting similarities to standard discussions of the relation of the Hesienberg and the Schrödinger views on quantum mechanics, such as Sakurai, 80-89. In my work on Feynman and Sakurai I found it convenient to do a detailed comparison of the two tables of contents: I would advise you to do the same with Sakurai or other texts. A context for your reflection here, and indeed for your entire effort, is provided by John Gribben and Mary Gribben, Richard Feynman. A Life in Science, Plume Penguin pb, 1998. "One of the strange features of quantum mechanics is that right from the moment it was invented (or discovered) in the mid-1920s, there were two completely different descriptions of the quantum world. One was
perhaps, you will whisker paw that conclusion within your spontaneous hunting bent of section 6.6 .7 of chapter 3 of Insight:" A Principle of Uncertainly. An axiomatic structure for statistical laws will involve an uncertainly principle. ${ }^{322}$ And so you may come to pause, paws, comeabout to pause, in a held and holding space-time that is not anyway out there, over "The Dependence of Amplitude on Position." ${ }^{33}$ You will come, contrastingly, to grip the dependence of space and time - think of their odd relations to Space and Time ${ }^{34}$ - on amplitude, aptitude, a grip that is a holding of a concrete and

Schrödinger's approach, based on waves; the other was Heisenberg's approach, based on particles. Both versions of quantum theory had been shown to be exactly equivalent.... Now Feynman had found a third approach to quantum mechanics, based on action .... But this approach never caught on. In universities around the world, even today, half a century after Feynman's insight, students are still taught classical mechanics on the old-fashioned way." (op. cit., 88-89) My suggestion in these two Joistings is that a student can profitably supplement the conventional texts with the Feynman approach given elementarily in FIII. There is, of course, a fuller challenge, as Carver Mead points out (Collective Electrodynamics. Quantum Foundations of Electromagnetism, MIT Press, 2000, xii-xiv). Chapter 21 of FIII has the seeds in it of another angle on the whole business, which could be profitably followed up in Mead's text. See also the final footnote here.
${ }^{32}$ Insight, 99[123]. But one has also to lift that text into the context both of chapter 16 of the book and the fuller concrete view of a meshing of probability theory with the concrete divergent entropic - and negentropic - energy-splicings that pattern fundamental dispersedness. Notes 14 and 15 above already raised this issue.
${ }^{33}$ The title of that of $\mathbf{F} 16$, but the pointing here, and in these final footnotes, is towards a massive sublation analogous to the sublation offered present economics by Lonergan's For A New Political Economy. Recall the comments earlier on the short quotation from F7. 16.11 ...
${ }^{34}$ The bow, in the later culture of the third stage of meaning, is to be an incarnate and luminous bow to the already-in-here- now of organic neurodynamics as the empirical residence of our organic journey. Place the problem, as far as you can, in the context of Insight chapter 19, section 7. But you must struggle towards being self-tastingly up-to-date. A help here are The Feynman Lectures, Volume 1, chapter 35, "Colour vision" and chapter 36, "The Mechanics of Seeing". You might even pick up on Scientific American, July 2006: "What Birds See": ""'Colour is not actually a property of light or of objects that reflect light. It is a sensation that arises within the brain" (p.72).
impossibly complex geometry, ${ }^{35}$ a geometry that bows to dispersedness as granting to our loneliness a mathematics of qualified continuity ${ }^{36}$ and a physics of history that doubts the existence of points and strings. ${ }^{37}$

[^7]${ }^{36}$ Recall notes 14 and 15 above. I would note the openness implied in my pointings here. After Aquinas there seems to have been little regard for the mediation by theory of metaphysics. So, for example, present scholasticism, and I include the scholasticism of Lonerganism, would not connect the struggle with the meaning of aether in the two volumes of Sir Edmund Whittaker, History of Theories of Aether and Electricity, (Harper, New York, pb,1960) with a push towards a richer grasp of the empirical residue, our empirical residence.
${ }^{37}$ Cantowers 42-81 (September 2005 - December 2008) were to have been an initial stab at this full cultural shift. Central to the above topic is Cantower 62: "Quantumchromodynamic Bags: No Strings Attached", but the fuller cultural shift would have been tackled in Cantower 53: "The International Search for Enlightenment". More on this issue in the following Joistings. But on the present issue there shall be no more from me. Circumstances and foundational orientation point me elsewhere. So, to previous notes that end with "and so on"(note 30) or "follow up" (note 32) I would add this final appeal to follow up and on.. What I have written here is very compact, nor have I ventured much out into the literature. So, for example, there is the question of non-point, non-line physics: strange mathematical and real topologies probably quite beyond the reachings of Whitehead and Grzegorczyk (see Loredana Biacino and Giangiacomo Gerla, "Connection Structures: Grzegorczyk's and Whitehead’s Definitions of Point," Notre Dame Journal of Formal Logic, 37 (1993), 431-39). Again, there are the issues I raised regarding complexity, probability, entropy, measurement, and a growing literature on problems of layered randomnesses. It seems good to end here on an open note of "follow up" "and so on", where such following-up needs the slowly emergent context of Lonergan's Standard Model of Effective Global Inquiry. I end by referring to a worthwhile context of nudges, Studies in the Sciences of Complexity. Volume VIII, edited by W.H.Zurek, AddisonWesley, 1990, contains various good articles but I limit myself here to a single page (385) of an article on zero-point energy (E.T.Jaynes, "Probability in Quantum Mechanics", 381-403) which brings us right back to the key problem from which we began. Jaynes writes of "the supreme self-confidence of the Copenhagen interpreters", of "Richard Feynman’s honesty to admit , 'Nobody knows how it can be that way'", and of "the failure of quantum theorists to distinguish quite different meanings of 'probability'. And so, On. If I were to select one other nudging page in that volume it would be 433, which gives "Figure 2. A schematic structure of the space of sets
of possible histories for the universe". The page is from the article by Murray Gell-Mann and James B.Hartle, "Quantum Mechanics in the Light of Quantum Cosmology" (415-457). Try lifting that muddle into the context of a schematic of emergent probability. And so, On. Sow on.


[^0]:    ${ }^{5}$ The essay (available on the website www.philipmcshane.ca) is well worth a visit, even if it seems quite a different context. The aim there was to throw light in the relationship between The Sketch of the task of interpretation (Insight, 579-81[602-3] and the canons of interpretation. I would note that the task of interpretation comes up first as a topic in chapter 5 of Insight(162-4 [186-88])
    ${ }^{6}$ Pierre Fermat (1601-1665) wrote this ("I have a marvelous solution to this problem, but the margin is too small for it") on the margin of Diophantus's Arithmetica. Andrew Wiles quotes it at the beginning of his 108-page solution to the problem: "Modular Elliptic Curves and Fermat's Last Theorem", The Annals of Mathematics, $2^{\text {nd }}$ Ser., Vol.141, 443-551.

[^1]:    ${ }^{7}$ Sakurai introduces this topic on page 118, but you need to venture into Feynman's book, Quantum Mechanics and Path Integrals, McGraw-Hill, 1965 (edited by J.A.R.Hibbs) to move towards its meaning and significance. A brief introduction to that meaning is given in Feynman's very fine lecture on "The Principle of Least Action" (FI, chapter 19, page 9). The S here is not, of course, the $S$ of an S-matrix mentioned on page 8 of $\mathbf{F 8}$, but it would be

[^2]:    ${ }^{13}$ Sakurai is good, working with the "Analogy with Polarized Light" (6-10) in getting the beginner towards "the main goal of this section: to introduce the idea that quantum-mechanical states are to be represented by vectors in an abstract complex of vector space" (10), but a verified real geometry demands a great deal more, a pointing that emerges in our final notes here. Feyman's reflections on the complexity of a classroom's radiation helps here ( See Volume II, chapter 18, pages 8-9) and you might pause over the fact that the twinkle in your eye last week is a light week on the road to distant stars.
    ${ }^{14}$ I am pointing here the discomforting topic of entropy, introduced by Feynman on the first page of F7: ".... Why does the atom radiate light? The answer has to do with entropy. When the energy is in the electromagnetic field, ther4 are so many different ways it can be - so many different places it can wander - that if we look for the equilibrium condition, we find that in the most probable situation ...." Feynman pauses over the problem of entropy earlier in these volumes are worth brooding over. See, Volume 1, chapter 44, section 6 "Entropy" and chapter 46, section 5, "Order and Entropy". In Volume 2 there is the magnificent pedagogical effort in chapter 19 on "The Principle of Least Action", climbing to his final note regarding a minimum for energy generation and of entropy generation. "....does the same principle of minimum entropy generation also hold when the situation is described quantum-mechanically? I haven't found out yet." Add the context, mentioned in at the end of the previous Joistings, of Sir Arthur Eddington’s remark (Space, Time and Gravitation, Harper and Row pb, 1959, 178):"We combine probabilities by multiplying; bot we combine the actions in two regions by adding; hence the logarithm of a probability is indicated. Further, since the logarithm of a probability is necessarily negative, we may identify action provisionally with minus the logarithm of the

[^3]:    ${ }^{16} \mathrm{We}$ come here to the heart of the problem. Instead of a Fermat type note, what I add are notes that point to tasks that are to lead the global community cyclically through and beyond Insight. I use above the odd expression actual-possibilities. It is as odd and as suggestive as Lonergan's use of the expression capacity-for-performance in chapter 15 of Insight. We are in the presence - luminous perhaps to the initiated - of the absence of the developed metaphysics that is to emerge eventually from that cyclic development of the hints of chapters 15 and 16 of Insight. Have a personal shot, for instance, at sublating Margenau's view, mentioned in note 21 below, of 'latent' quantities in terms of the distinction between primary and secondary determinations mentioned, without development, in Insight chapter 16. Throw in Feynman's Path Integral approach to bring you - and perhaps the community if it listens to you - closer to form's actual dispersedness in an elusive geometry. The full heuristic of that geometry should include a grip on the character of "diverging conditions" (see Insight + Randomness, Statistics and Emergence) and the ground of entropy in the dynamics of the cosmos. We are back in the context pointed to in note 14.
    ${ }^{17}$ There are broad problems here, but it is best to keep the focus "small" as Feynman does. Think, then, in terms of the two-state systems that he considers. For example, start with the two equations of chapter 8 , labeled (8.43), giving rates of change of two dC/dt in terms of $\mathrm{H}_{11}$, $\mathrm{H}_{12}, \mathrm{H}_{21}, \mathrm{H}_{22}$. (8.52) and (8.53) give you solutions to these that enable you to think more definitely about aptitudes, and about the peculiar "probability suggestions", cos $\mathrm{At} / \mathrm{h}, \sin \mathrm{At} / \mathrm{h}$, that come out of all this. You can follow up particular versions of the two equations and there solutions in later chapters of Feynman e.g. equations (9.36). You can then ask about the character of the resulting functions in comparison with the usual functions of traditional probability theory: are these functions more projections belonging to strange spaces than the usual Gaussian etc functions? So you find yourself back with the questions posed in note 32 of Joistings 24.

[^4]:    ${ }^{23}$ The (about) ${ }^{3}$ refers to the to-be-developed third order consciousness of the Tower community ( see above, note 9). Membership in that community would "cajole, force"(Insight, 398[423]) the adult growth towards the "come about". No harm, now in repeating that challenge here: give you a chance to see how you measure up to its startling unrealism. "So it comes about that the extroverted subject visualizing extension and experiencing duration give place to the subject oriented to the objective of the unrestricted desire to know and affirming beings differentiated by certain conjugate potencies, forms, and acts grounding certain laws and frequencies"(Insight, 514[537]). Cantower 9 could be a help in your first decade of that struggle.

[^5]:    ${ }^{24}$ I am recalling here a song-poem of Patrick Kavanagh, which expresses a favorite theme of his: "If ever you go to Dublin Town in a hundred years or so". I mark the date, from now, as 2111.
    > ${ }^{25}$ The previous notes are obviously a brief recall, but it has been my central topic since I enlarged on its relevance to musicology in 1969 (See chapter 2 of The Shaping of the Foundations). Chapter 5 of Method in Theology is the obvious place to start, but with a push to taste the eightfold division as an elementary global need. That elementary need is pushed forward pedagogically beginning with chapter one of my recent Method in Theology and Botany.

[^6]:    ${ }^{26}$ The dynamics and illusions of this optimism are massively complex topics, lifting issues of haute vulgarization, popularization and pedagogy into later treatises on organic adult human growth. Those treatises are to include a dialectic analysis of Lonergan's thus-warped Opera Omnia.
    ${ }^{27}$ Respectively chapter 7 and chapter 16 of FIII..
    ${ }^{28}$ Feynman's strategy, which runs through the book, is very seriously helpful. You might think of it as somewhat paralleled by the two-body problem (or Fermat's theorem in relation to the power of 2 !). See note 17 above.

[^7]:    ${ }^{35}$ You can get a taste of the move towards and achievement of such a geometry in Lochlainn O'Raifeartaigh, The Dawning of Gauge Theory, Princeton, 1997) and Group Structure of Gauge Theory, Cambridge University Press, 1986). More elementarily there is Ian Lawrie, A Unified Grand Tour of Theoretical Physics, chapter 8, "Forces, Connections and Gauge Fields". But the personal self-tasting work needs the bracketing of the bracketing footnotes here, notes 34 and 36 .

